Canonical Gravity, Diffeomorphisms and Objective Histories

Joseph Samuel
Raman Research Institute
Bangalore 560 080, INDIA.

email:sam@rri.ernet.in

Abstract

We clarify the meaning of diffeomorphism invariance in constrained Hamiltonian formulations of General Relativity. We observe that, if a constrained Hamiltonian formulation derives from a manifestly covariant Lagrangian, the diffeomorphism invariance of the Lagrangian results in the following properties of the constrained Hamiltonian theory: the diffeomorphisms are generated by constraints on the phase space so that a) The algebra of the generators reflects the algebra of the diffeomorphism group. b) The Poisson brackets of the basic fields with the generators reflects the space-time transformation properties of these basic fields. This suggests that in a purely Hamiltonian approach the requirement of diffeomorphism invariance should be interpreted to include b) and not just a) as one might naively suppose. Giving up b) amounts to giving up objective histories, even at the classical level. This observation has implications for Loop Quantum Gravity which are spelled out in a companion paper.

1 Introduction

The diffeomorphism invariance of General Relativity presents both conceptual and technical problems[1] for quantisation. At the conceptual level, it leads to deep questions about the nature of time, observables and the interpretation of quantum theory. At the technical level, diffeomorphism invariance leads to constraints on the classical phase space [1], which in a quantum theory, must be imposed on physical states. Solving these constraints has occupied much of the effort in the canonical approach to quantum gravity. Several constrained Hamiltonian formulations (CHFs) of General Relativity exist today, each with its own following. It remains to be seen which of these formulations will be the most advantageous in the approach to quantum General Relativity.

This paper seeks to clarify the meaning of diffeomorphism invariance in a classical, constrained Hamiltonian Theory. Given a constrained theory, how does one test for diffeomorphism invariance? The answer to this question involves a subtlety, on which we focus in this paper. Our strategy in addressing this question will be to start with CHF's which we *know* are diffeomorphism invariant: those that are derived by a Legendre transformation from a manifestly covariant Lagrangian. We will then notice that the resulting constrained Hamiltonian formulation satisfies certain conditions as a consequence of the diffeomorphism invariance of the Lagrangian starting point. We will explicitly spell out these conditions and use these as a criterion for testing for diffeomorphism invariance even when a Lagrangian starting point is not available. For example many currently popular CHFs of General Relativity [2, 3] are derived by making canonical transformations on the phase space; they are entirely Hamiltonian in spirit and abandon the Lagrangian approach. The diffeomorphism invariance of such formulations is open to doubt and has to be checked. The purpose of this paper is to clarify how this can be done, in a purely Hamiltonian framework.

The paper is organised as follows: In section II, we recapitulate some known results about the gauge symmetries of Lagrangian systems and show how these symmetries manifest themselves in a Hamiltonian framework. In section III we illustrate these general results using familiar examples like the ADM formalism, gravity in 2+1 dimensions and Ashtekar's extended phase space construction (EPS). In section IV, we distinguish between strong and weak diffeomorphism invariance of a CHF and bring out an analogy with a much simpler situation: relativistic particle dynamics. Section V is a concluding discussion.

2 Symmetries of Singular Lagrangian systems

Consider a dynamical system with configuration manifold $\mathcal{Q}$ on which local co-ordinates are $q^r, r = 1..n$. The tangent bundle over $\mathcal{Q}$ is $T\mathcal{Q}$ and the Lagrangian $L(q, \dot{q})$ is a real valued function on $T\mathcal{Q}$. The Lagrangian L defines a map from $T\mathcal{Q}$ to the cotangent bundle $T^*\mathcal{Q}$ defined locally by $p_r = \frac{\partial L}{\partial \dot{q}^r}$. In the cases of interest in this paper, the Lagrangian L is singular, *i.e*, the Legendre map $\Phi : T\mathcal{Q} \to T^*\mathcal{Q}$ is not onto. Its image Σ is a proper subset of $T^*\mathcal{Q}$: $\Phi(T\mathcal{Q}) = \Sigma \subset T^*\mathcal{Q}$ and there are constraints on the phase space. Such situations are dealt with in Dirac's theory of constrained systems [1]. One iteratively demands preservation of the constraints and this leads, in general, to more constraints. The algorithm terminates when no new constraints emerge. The total set of constraints are divided into first and second class and we suppose that the second class constraints are eliminated by passage to the Dirac bracket. An elegant way to do this is to use the Bergmann-Komar starring procedure[4]. One simply replaces all phase space functions by their starred counterparts. After the Dirac constraint analysis ends, one has a constrained Hamiltonian formulation which has the following ingredients: i) the basic variables (or fields, in a field theory) are (q^r, p_r) which span the phase space obeying commutation relations[5]. ii) a physical interpretation for q^r and p_r that derives from their definitions as functions of q and $\dot{q}$. iii) a set of constraints which emerge from the constraint analysis. iv) A Hamiltonian function on the phase space, which generates the dynamics and preserves the constraints. The Hamiltonian is arbitrary to the extent of a primary first class constraint[6].

Let us recapitulate a few known results [7, 8, 9] about the continuous symmetries of singular Lagrangian systems. Let $S^r(q, \dot{q}, t)$ be a symmetry transformation. By this we mean that the change $\delta_S L$ in the Lagrangian under the changes $\delta_S q^r = \epsilon S^r(q, \dot{q}, t)$, $\delta_S \dot{q}^r = \epsilon \dot{S}^r$ in $(q^r, \dot{q}^r)$ is given by a total divergence:

$$\delta_S L = \epsilon \frac{dF(q, \dot{q}, t)}{dt}. \tag{1}$$

(Note that in (1) we do not use the Euler-Lagrange equations, the accelerations are unrestricted.) From (1) it follows that on solutions to the equations of motion, the quantity $G_{\mathcal{L}}(q, \dot{q}, t) := \frac{\partial L}{\partial \dot{q}^r} S^r - F$ is conserved as a result of Nöther's theorem. (1) also implies that

$G_{\mathcal{L}}(q, \dot{q}, t)$ is projectable [8, 9] under the Legendre map and therefore can be expressed as the pull back of a function on Σ : $G_{\mathcal{L}} = \Phi^* G$, . In general, the symmetry vector field $X_S := S^r \frac{\partial}{\partial q^r} + \dot{S}^r \frac{\partial}{\partial \dot{q}^r}$ (which is defined on $T\mathcal{Q}$ by using the equations of motion, or more briefly, the dynamics Δ) is *not* Φ projectable [10]. The vertical part of X_S projects down to zero, and the horizontal part can be expressed in the form

$$\begin{aligned} \delta_S q^r &= \epsilon(\frac{\partial G}{\partial p_r} + u^\rho \frac{\partial \phi_\rho}{\partial p_r}) \\ \delta_S p_r &= -\epsilon(\frac{\partial G}{\partial q^r} + u^\rho \frac{\partial \phi_\rho}{\partial q^r}) \end{aligned} \tag{2}$$

where $\phi_\rho(q, p)$ are the primary constraints [6]. The functions u^ρ are functions on $T\mathcal{Q}$, which are not in general projectable under Φ. The non-projectability of X_S has been isolated in the functions u^ρ, which depend not only on the phase space variables (q^r, p_r), but also the "unsolved velocities" v^ρ. As the Dirac analysis proceeds, the dynamics, and with it the symmetry vector field (which depends on the dynamics), gets more sharply determined [9]. From the basic identity (1) it follows that the symmetry is "compatible" with the dynamics throughout the constraint analysis: if the dynamics preserves constraints, so does the symmetry. The symmetry generator of X_S is $\mathcal{G}_S = G + u^\rho \phi_\rho$, where we notice that although the $u^\rho(q, p, v)$ are not phase space functions, we never need to know the Poisson bracket of u^ρ with anything, since these terms vanish on the constraint surface. Symmetries of the Lagrangian translate into the following properties of the constrained Hamiltonian formulation, which hold *on shell*, (i.e, modulo the equations of motion):

a) The Lie algebra of the symmetry group is reflected in the bracket relations of the symmetry generators $\mathcal{G}$. If S_1 and S_2 are symmetries on $T\mathcal{Q}$ with commutator $[S_1, S_2]$:

$$\{\mathcal{G}_{S_1}, \mathcal{G}_{S_2}\} = \mathcal{G}_{[S_1, S_2]}. \tag{3}$$

b) The basic variables q^r and p_r are functions on $T\mathcal{Q}$ and transform in a definite manner under the symmetry tranformation S. This transformation property [12] is reflected in the bracket relations between these basic variables and $\mathcal{G}_S$.

$$\begin{aligned} \delta_S q^r &= \epsilon\{q^r, \mathcal{G}_S\} \\ \delta_S p_r &= \epsilon\{p_r, \mathcal{G}_S\}, \end{aligned} \tag{4}$$

where $\{,\}$ refers to the Dirac bracket resulting from elimination of second class constraints (if any).

In the rest of this paper we apply these general considerations to the case of interest to us. We consider constrained Hamiltonian formulations of General Relativity and the symmetry of interest is diffeomorphism invariance. In this case, as is well known, the generators $\mathcal{G}_S$ are a linear combination of constraints. The criteria listed above can be used to test for invariance even in a purely Hamiltonian framework i.e, even when a Lagrangian is absent. Below, we will slightly weaken them to allow for the possibility that they are satisfied modulo gauge [13] transformations[14].

3 Diffeomorphism Invariant Formulations

We now examine some constrained Hamiltonian formulations of diffeomorphism invariant theories to see that they do indeed satisfy *both* the conditions listed above. All of these formulations are derived from diffeomorphism invariant Lagrangians. Let $(\mathcal{M}, g_{\mu\nu}), \mu = 0, 1, 2, 3$ be a space-time manifold, topologically $\mathcal{S} \times I\!\!R$. To simplify matters, we will assume that $\mathcal{S}$ has no boundary so that we don't need to keep track of spatial boundary terms. We are also interested only in infinitesimal diffeomorphisms and deal entirely with the Lie Algebra rather than the Lie Group of diffeomorphisms.

From the general discussion of the last section, we expect that the CHF's will satisfy the conditions a) and b) above. We write down these conditions explicitly in a few concrete cases and note that they *are* satisfied.

ADM formulation: The ADM formulation consists of the following ingredients: the basic variables are $(q_{ab}, \tilde{\pi}^{ab})$, which are canonically conjugate. q_{ab} is the pullback of the space-time metric to a spatial slice $\mathcal{S}$ and $\tilde{\pi}^{ab}$ is its conjugate momentum. The basic variables $(q_{ab}, \tilde{\pi}^{ab})$ are subject to the Hamiltonian constraint

$$\tilde{\mathcal{H}} = \frac{1}{\sqrt{q}}(\tilde{\pi}^{ab}\tilde{\pi}_{ab} - \frac{1}{2}\tilde{\pi}^2) - \sqrt{q}{}^3R \approx 0$$

and the spatial diffeomorphism constraint

$$\tilde{\mathcal{H}}^b = D_a\tilde{\pi}^{ab} \approx 0$$

where D is the covariant derivative compatible with the three-metric q_{ab}.

If ξ^μ is a vector field on space-time $\mathcal{M}$ generating an infinitesimal diffeomorphism we can decompose it normal and tangential to $\mathcal{S}$:

$$\xi^\mu = N\hat{n}^\mu + N^\mu,$$

where $\hat{n}^\mu$ is the unit normal to $\mathcal{S}$. The combination of constraints generating this diffeomorphism is

$$C(\xi) = \int_{\mathcal{S}} (N\tilde{\mathcal{H}} + N^a\tilde{\mathcal{H}}_a).$$

If ξ_1 and ξ_2 are two vector fields and $[\xi_1, \xi_2]$ their Lie bracket, then generators $C(\xi_1)$ and $C(\xi_2)$ satisfy the relation

$$\{C(\xi_1), C(\xi_2)\} = C([\xi_1, \xi_2]),$$

i.e., the Poisson brackets of the generators of diffeomorphism reflect the Lie algebra of the diffeomorphism group. Thus condition, (a) is satisfied as one would expect.

But this is not all: the condition (b) also holds, as one would expect from the general analysis of the last section. The basic variables of the theory are $(q_{ab}, \tilde{\pi}^{ab})$ and they have definite space-time meaning: q_{ab} is the pull-back of the space-time metric to a spatial slice $\mathcal{S}$. By using Hamilton's equations of motion we see that $\tilde{\pi}^{ab}$ is algebraically related to the extrinsic curvature of $\mathcal{S}$. Since the basic fields have a clear space-time meaning, they have a definite transformation property under space-time diffeomorphisms. For example, under an infinitesimal diffeomorphism generated by a vector field ξ^a tangent to $\mathcal{S}$ ($a = 1, 2, 3$ is a spatial index), we expect

$$\delta q_{ab}(\mathrm{space-time}) = (D_a\xi_b + D_b\xi_a).$$

If ξ is normal to $\mathcal{S}$, $\xi^\mu = N\hat{n}^\mu$ we expect

$$\delta q_{ab}(\mathrm{space-time}) = \mathcal{L}_\xi q_{ab} = NK_{ab},$$

where K_{ab} is the extrinsic curvature of $\mathcal{S}$.

One can also compute the change in the basic variables by taking their Poisson brackets with the diffeomorphism generator $C(\xi)$:

$$\begin{aligned} \delta_\xi q_{ab}(\mathrm{canonical}) &= \{q_{ab}, C(\xi)\}, \\ \delta_\xi \tilde{\pi}^{ab}(\mathrm{canonical}) &= \{\tilde{\pi}^{ab}, C(\xi)\}. \end{aligned} \tag{5}$$

The condition (b) is satisfied in the ADM formalism since [15] as follows from Hamilton's equations

$$\begin{aligned} \delta_\xi q_{ab}(\mathrm{space-time}) &= \delta_\xi q_{ab}(\mathrm{canonical}) \\ \delta_\xi \tilde{\pi}^{ab}(\mathrm{space-time}) &= \delta_\xi \tilde{\pi}^{ab}(\mathrm{canonical}). \end{aligned}$$

2+1 Palatini gravity: The next example we consider is gravity in 2+1 dimensions in its Palatini formulation. The basic fields are e^I_μ and A^{IJ}_μ; $\mu = 0, 1, 2$ is a tangent space index and $I = 0, 1, 2$ is an internal Minkowski index. e^I_μ is a triad and A^{IJ}_μ an $SO(2, 1)$ connection. The action is given by

$$I = \frac{1}{2} \int e_I \wedge F^I,$$

where $F = dA + A \wedge A$ in the notation of differential forms. A standard constraint analysis leads to the following Hamiltonian formulation: The basic variables are the canonically conjugate pair $(\tilde{e}^{I\,a} := \tilde{\eta}^{ab} e^I_b, A^I_a)$, where a is a spatial index. The constraints of the theory are

$$\begin{aligned} F^I &= 0 \\ G^I &= \mathcal{D} \wedge e^I = 0, \end{aligned}$$

where it is understood that these two-forms are pulled back to a spatial slice $\mathcal{S}$. Diffeomorphism are generated by combinations of constraints. If ξ^μ is a vector field on $\mathcal{M}$,

$$C(\xi) = \int_{\mathcal{S}} (\xi^\mu e^I_\mu F_I + \xi^\mu A^I_\mu G_I)$$

generates a pure diffeomorphism on the basic variables. It is easily checked, using the $ISO(2, 1)$ algebra satisfied by the constraints that both (a) and (b) above are satisfied on shell (using the equations of motion).

Extended phase space construction (EPS): As a last example, we consider the extended phase space of Ashtekar. This CHF was originally arrived at by Ashtekar [16] by extending the ADM phase space to incorporate triads. This example is instructive because it can also be derived [16, 17, 18] from a manifestly covariant Lagrangian *by fixing the "time gauge"*. This example will illustrate how internal gauge fixing interacts with diffeomorphism invariance. As we will see, because of the gauge fixing a) and b) are not satisfied as they stand but they *are* satisfied modulo $SO(3)$ gauge.

Let us start with the following action principle. The basic fields are e^I_μ, A^{IJ}_μ, where e^I_μ is a tetrad field and A^{IJ}_μ an $SO(3, 1)$ connection field. The action is

$$I = \frac{1}{2} \int e^I \wedge e^J \wedge F^{KL} \epsilon_{IJKL}, \tag{6}$$

where we use differential form notation and $F = dA + A \wedge A$. A straight forward Legendre transformation [16] results in the following CHF. The basic conjugate variables are $(A_a, \tilde{\alpha}^a)$

where

$$\tilde{\alpha}^a_{IJ} = \tilde{\eta}^{abc} e_{bI} e_{cJ}. \tag{7}$$

These variables are subject to the constraints

$$G_{IJ} = \mathcal{D}_a \tilde{\alpha}^a_{IJ} \approx 0 \tag{8}$$

$$V_a = Tr \tilde{\alpha}^b F_{ab} \approx 0 \tag{9}$$

$$S = Tr \tilde{\alpha}^a \tilde{\alpha}^b F_{ab} \approx 0 \tag{10}$$

$$\phi^{ab} := \epsilon^{IJKL} \tilde{\alpha}^a_{IJ} \tilde{\alpha}^b_{KL} \approx 0 \tag{11}$$

$$\chi^{ab} := \epsilon^{IJKL} \tilde{\alpha}^{cM}_I \tilde{\alpha}^{(a}_{MJ} (\mathcal{D}_c \tilde{\alpha}^{b)})_{KL} \approx 0. \tag{12}$$

Of these the last two (11, 12) are second class. Let us suppose these second class constraints to be formally eliminated by passing to the Dirac bracket. No gauge fixing has been done so far and it follows from the general theory summarised in the last section that the Hamiltonian formulation above satisfies a) as well as b).

(11) implies [16] that $\tilde{\alpha}_{IJ}$ is of the form $\tilde{E}^a{}_{[I} n_{J]}$ for *some* internal vector n_J. Let us now impose the "time" gauge, i.e., pick n_I to have the standard form $\overset{\circ}{n}{}^I = (1,0,0,0)$. This corresponds to choosing e^0 to be normal to the spatial slice $\mathcal{S}$. One is, of course, at liberty to make this gauge choice. In order to enforce this gauge choice, we need to impose a constraint

$$\chi^I = n^I - \overset{\circ}{n}{}^I \approx 0. \tag{13}$$

This constraint breaks the $SO(3,1)$ gauge generated by the Gauss law constraint (8) down to $SO(3)$. The "Boost part"

$$B_I = G_{IJ} \overset{\circ}{n}{}^J \tag{14}$$

of (8) does not commute with (13) and in fact (B_I, χ^I) form a second class set. If one eliminates this second class set one arrives at EPS. Writing i, j instead of I, J for indices orthogonal to $\overset{\circ}{n}{}^I$, we find that basic variables of EPS are $(\tilde{E}^a{}_i, K_a{}^i)$ which are canonically conjugate and have the space-time interpretation of densitised triad and extrinsic curvature respectively. The constraints of the theory are:

$$\begin{aligned}
\epsilon_{ijk} K^j_a \tilde{E}^{ak} &\approx 0 \\
D_a [\tilde{E}^a_k K^k_b - \delta^a_b \tilde{E}^c_k K^k_c] &\approx 0 \\
\sqrt{q} R + \frac{2}{\sqrt{q}} \tilde{E}^{[a}_i \tilde{E}^{b]}_j K^i_a K^j_b &\approx 0,
\end{aligned}$$

where D_a is the covariant derivative associated with q_{ab} and R, its scalar curvature.

Are conditions a) and b) satisfied in the gauge fixed theory? Diffeomorphisms that displace $\mathcal{S}$ normal to itself will in general, spoil the "time gauge". In order to restore the "time gauge" (and this is the BK starring procedure of passing to Dirac brackets) one has to add some definite linear combination of B_I to the diffeomorphism generator. As a result, (since the commutator of two boosts is a rotation) the diffeomorphism algebra closes only up to $SO(3)$ gauge rotations. In the same way, (b) is only satisfied up to $SO(3)$ gauge rotations. We describe this theory as satisfying a) and b) (mod $SO(3)$ gauge). The lesson to be learned from this example is that if one derives a Hamiltonian formulation from a Lagrangian and fixes gauge in the derivation, the resulting Hamiltonian formulation is diffeomorphism invariant(modulo gauge).

4 Strong and Weak Diffeomorphism Invariance

It is clear from these examples that diffeomorphism invariance in the Hamiltonian framework means *more* that realizing the diffeomorphism algebra as constraints. It is also necessary that under the action of the diffeomorphism generators, the basic variables must transform as expected from their space-time interpretation. We will refer to a CHF which satisfies the first condition (a) as weakly diffeomorphism invariant. A theory that also satisfies (b) is called strongly diffeomorphism invariant. It is clear that before we can test a CHF for diffeomorphism invariance, the space-time meaning of the basic variables has to be declared, since condition (b) explicitly needs this knowledge. Indeed unless the space-time meaning of the basic variables is declared the CHF does not make any contact with the real world!

To better understand the meaning of Strong Diffeomorphism invariance, it is useful to consider a simpler but analogous situation: classical relativistic particle dynamics [19, 20]. Direct interactions between N relativistic particles in Minkowski space can be described by mathematical models which are constrained Hamiltonian formulations. The models are defined as follows: the basic variables are $({x_a}^\mu, p_{a\mu}), (a = 1..N, \mu = 0, 1, 2, 3)$, where a is particle index (for the duration of this section) and μ a Minkowski space-time index. One can define the system by imposing constraints. The constraints are needed to reduce the phase space degrees of freedom from $8N$ to $6N$, which is the right number for N particles. The symmetry of interest here is the Poincare group. We will say that a model is Poincare invariant if the following conditions hold:

a)There exist 10 functions (one for each of the Poincare generators) on the phase space whose Dirac brackets reflect the Lie Algebra of the Poincare group.

b) The Dirac brackets between the basic variables $(x^{\mu}{}_a, p_{a\mu})$ and the Poincare generators reflect the space-time transformation properties of the basic variables.

As was first pointed out by Pryce [21], Poincare invariance means *both* a) and b) and not just a). Early work in this area [22] explicitly gave up the condition b). In these models [22], particle world lines would depend on the Lorentz frame of the observer. (To clarify this point, it is not just the same world line viewed from different Lorentz frames, but *different* world lines.) This amounts to giving up the objectivity of world lines, or particle histories, which is unacceptable, since classically, particle world lines can be experimentally measured [23].

Returning to our problem in canonical gravity, a CHF which is only weakly diffeomorphism invariant suffers from the following feature: Given a space-time history (a solution of the field equations), one can slice it up in many ways in a 3+1 formalism. Conversely, given initial data and particular slicing one can evolve the initial data and produce a "history" by "stacking" the spatial slices in temporal order[24]. In theories where b) is given up the "history" which is produced *depends on the slicing.* This means that the history has no objective reality. One would of course like measureable quantities to have an objective meaning (independent of slicing). One should therefore be aware of which fields in a theory are objectively real. For example, in the EPS, the fields q_{ab} and K_{ab} (which are $SO(3)$ gauge invariant) *do* have objective reality. But the basic fields in the formulation $(\tilde{E}^a{}_i, K_a{}^i)$ do not. They are only defined modulo $SO(3)$ gauge.

5 Conclusion

We have shown that for a constrained Hamiltonian formulation of gravity to be diffeomorphism invariant there must be diffeomorphism generators on the phase space so that a) the generators reflect the algebra of the diffeomorphism group in their brackets and b) the space-time interpretation of the basic fields is reflected in their brackets with the diffeomorphism generators. These conditions are automatically satisfied by CHFs which derive from a covariant Lagrangian. Notice that a covariant Lagrangian automatically gives us space-time interpretations for all the phase space variables appearing in the Hamiltonian formulation. In a purely Hamiltonian approach one has to not only prescribe the basic variables, their brackets, and the constraints, but also give a space-time interpretation for the basic variables. Unless this is done, it is not possible to physically interpret the Hamiltonian system. If the PB of the diffeomorphism generator with a phase space variable does not reflect its space-time interpretation, one loses a space-time interpretation for that variable even at the classical level.

The Diffeomorphism invariance of the theory can only be decided after the space-time interpretation of the basic variables has been declared (since condition b) explicitly needs this knowledge). Indeed, unless the space-time interpretation of the basic variables is declared, the CHF is not even fully defined, because it doesn't make contact with the real world. A CHF may be diffeomorphism invariant with one interpretation and not invariant with another space-time interpretation of the basic variables. An example of this phenomenon is discussed in [25]. Barbero's Hamiltonian formulation of General Relativity *is* strongly diffeomorphism invariant with one space–time interpretation of the basic variables, but not with the space–time "gauge field interpretation" that one might prefer.

In the EPS model, the Lie Algebra of the Diffeomorphism group is not a subalgebra of the constraint generators, but appears as a quotient. We arrived at conditions (3,2) by assuming that the symmetry group of interest was a *subgroup* of the total Lagrangian symmetry group. The known Lagrangian formulations of General Relativity all have the property that the diffeomorphism group is a subgroup. One can slightly relax this assumption and allow for "twisted products", where the diffeomorphism group only appears as a quotient. The situation then is very similar to the EPS formulation, where the diffeomorphism Lie Algebra only closes modulo gauge.

One may object that one should not demand that the basic variables be objectively defined in space-time, since they are not "observables" in the Dirac sense. This objection is easily met: it is easy to construct "observables" from the basic variables by using a device explained in [26]. Although q_{ab} is not an "observable", the distance between invariantly specified events is an "observable". E.g, one can locate an event as the intersection of two particle world lines or (in the absence of matter) as a point where four scalars constructed from the gravitational field [27] vanish. If a CHF is strongly diffeomorphism invariant in the sense of this paper, such "observables" do have an objective meaning. Otherwise, the answer predicted by the CHF could depend on slicing. A CHF which violates strong diffeomorphism invariance classically should be rejected as an unsuitable starting point for building a quantum theory.

Acknowledgement: It is a pleasure to thank Richard Epp, B.R. Iyer, Sukanya Sinha and Madhavan Varadarajan for extended discussions.

References

[1] Dirac, P A M, 1964 *Lectures on Quantum Mechanics* and references therein.
Arnowitt, R, Deser,S and Misner C W 1962, "The dynamics of General Relativity" in *Gravitation: An Introduction to Current Research* ed. L. Witten, (New York, Wiley) and references therein; For more references see
Kuchar, K V, 1992 *Canonical Quantum Gravity*, talk delivered at GRXII, Cordoba, Argentina.

[2] Ashtekar A (1986) *Phys. Rev. Lett.* **57** 2244
AshtekarA (1987) *Phys. Rev.* **D36** 1587.

[3] Barbero J F *Phys. Rev.* **D51** 5507.

[4] Bergmann P G and Komar A, 1960 *Phys. Rev. Lett.* **4, 432**.

[5] these may not be canonical if some second class constraints have been eliminated

[6] We do not follow Dirac's suggestion of "extending" the Hamiltonian by adding arbitrary combinations of the secondary first class constraints to it, since we wish to stick with the Lagrangian starting point. This equally means that we do not "extend" the symmetry vector field in a similar manner.

[7] Mukunda N, 1976 *Ann. Phys.* **99** 408.
Mukunda N, 1980 *Physics Scripta* 21 783.

[8] Kamimura K 1982 Singular Lagrangian and constrained Hamiltonian Systems *Il Nuovo Cimento* **68B** 33

[9] Marmo G, Mukunda N and Samuel J 1983, Dynamics and Symmetry for Constrained Systems: A Geometrical Analysis *Il Nuovo Cimento* **6** No. 2.

[10] The non-projectability of a symmetry transformation has also been recently remarked in [11]. While these papers too address the question of interplay between gauge symmetries and diffeomorphism invariance, their motivation is different from ours: they seek to find combinations of diffeos and gauge which *are* Φ projectable. Our interest here is in *pure* diffeo's , which in general are not projectable.

[11] Pons, J M, Salisbury D C, Shepley L C , gr-qc 9912085, gr-qc 9912086, gr-qc 9912087.

[12] It is actually true, since (q^r, p_r) span the phase space, that b) implies a). The converse, of course, is not true: a) does not imply b).

[13] In this paper, we reserve the word 'gauge' to mean "internal" gauge. Diffeomorphisms will not be referred to as 'gauge' transformations.

[14] In this case the diffeomorphism group is twisted with the internal gauge group. This point is discussed further in the concluding section.

[15] Carlip S (1998) *Quantum Gravity in 2+1 Dimensions*, CUP, Cambridge.

[16] Ashtekar A (1991) (notes prepared in collaboration with R. Tate), Lectures on Non-perturbative canonical gravity, World Scientific, Singapore.

[17] Ashtekar A, Balachandran A P and Jo S (1989), **A4** 1493.

[18] Romano, J. (1991) *Connection dynamics Vs Geometrodynamics)* Ph.D Thesis, Syracuse University.

[19] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[20] Currie, D G. Jordan T and Sudarshan E C G (1963) *Rev. Mod. Phys.* **35** 350;
Komar A (1978) *Phys.Rev.* **D18** 1881, 1887, 3617;
(1981) *Phys. Rev.* **D24** 2330;
Rohrlich F (1979) *Ann. Phys.* **117** 292;
(1979) *Physica* **968** 290;
Todorov I T (1971) *Phys. Rev.* **D3** 2351
Molotkov V V and Todorov I T (1981) *Comm. Math. Phys.* **79** 111. Samuel J (1982) *Phys. Rev.* **D26** 3475, 3482. Mann, R A (1974) *The classical dynamics of point particles: Gallilean and Lorentz Relativity*, Academic Press NY.

[21] Pryce M H L (1948) *Proc. Roy. Soc. London* **A195**, 62.

[22] Bakamjian B and Thomas L H (1952) *Phys. Rev.* **92**, 1300.

[23] There are models in which these conditions are satisfied modulo reparametrisation gauge. This is quite acceptable since, it does not compromise the objectivity of particle World lines. All that happens is that the World line is reparametrised under a Poincare transformation.

[24] Misner C W, Thorne K S and Wheeler J A (1970) *Gravitation* W.H. Freeman and Co. NY.

[25] J. Samuel, 'Is Barbero's Hamiltonian Formulation a Gauge Theory of Gravity?', preprint.

[26] Gaul M and Rovelli C (1999) *Loop Gravity and the meaning of Diffeomorphism invariance* gr-qc 9910079.

[27] Bergmann P G and Komar A (1972) *Int. Journ. Theor. Phys.* **5**15.